When you are working on this material, you must work entirely on your own. You may use the book or your notes. But you may not work with other people, either in the class our outside of it, or use other books or the Internet. If you have any questions, email me.

1. Find the eigenvalues and eigenspaces for this matrix.

$$\begin{pmatrix} -2 & 1 \\ 12 & -3 \end{pmatrix}$$

Solution: The characteristic equation comes from this determinant equation.

$$0 = \begin{vmatrix} -2-x & 1 \\ 12 & -3-x \end{vmatrix} = (-2-x)(-3-x) - (1)(12) = x^2 + 5x - 6 = (x-1)(x+6)$$

So the eigenvalues are $\lambda_1 = 1$ and $\lambda_2 = -6$.

For the eigenspaces consider this linear system.

$$\begin{aligned} (-2-x)b_1 + \qquad\qquad b_2 &= 0 \\ 12b_1 + (-3-x)b_2 &= 0 \end{aligned}$$

Find the eigenspace associated with $\lambda_1 = 1$ by plugging in $x = 1$.

$$\begin{aligned} -3b_1 + \quad b_2 &= 0 \\ 12b_1 + -4b_2 &= 0 \end{aligned} \xrightarrow{4\rho_1+\rho_2} \begin{aligned} -3b_1 + b_2 &= 0 \\ 0 &= 0 \end{aligned}$$

Leading is b_1 and free is b_2.

$$V_1 = \{ \begin{pmatrix} (1/3)b_2 \\ b_2 \end{pmatrix} \mid b_2 \in \mathbb{C} \} = \{ \begin{pmatrix} 1/3 \\ 1 \end{pmatrix} \cdot b_2 \mid b_2 \in \mathbb{C} \}$$

Find the eigenspace associated with $\lambda_2 = -6$ by plugging in $x = -6$.

$$\begin{aligned} 4b_1 + \ b_2 &= 0 \\ 12b_1 + 3b_2 &= 0 \end{aligned} \xrightarrow{-3\rho_1+\rho_2} \begin{aligned} 4b_1 + b_2 &= 0 \\ 0 &= 0 \end{aligned}$$

Leading is b_1 and free is b_2.

$$V_{-6} = \{ \begin{pmatrix} (-1/4)b_2 \\ b_2 \end{pmatrix} \mid b_2 \in \mathbb{C} \} = \{ \begin{pmatrix} -1/4 \\ 1 \end{pmatrix} \cdot b_2 \mid b_2 \in \mathbb{C} \}$$