1. Find the inverse of this matrix.

$$\begin{pmatrix} 3 & -1 \\ -2 & 2 \end{pmatrix}$$

Solution: Using the formula

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

we get this.

$$\begin{pmatrix} 3 & -1 \\ -2 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 3/4 & -1/4 \\ -1/2 & 1/2 \end{pmatrix}$$

Use it to solve these systems.

a)

$$\begin{aligned} 3x - y &= 5 \\ -2x + 2y &= -1 \end{aligned}$$

Solution: Multiply both sides of the matrix equation by the inverse.

$$\begin{pmatrix} 3/4 & -1/4 \\ -1/2 & 1/2 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ -2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3/4 & -1/4 \\ -1/2 & 1/2 \end{pmatrix}\begin{pmatrix} 5 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9/4 \\ 7/4 \end{pmatrix}$$

b)

$$\begin{aligned} 3x - y &= -4 \\ -2x + 2y &= 0 \end{aligned}$$

Solution:

$$\begin{pmatrix} 3/4 & -1/4 \\ -1/2 & 1/2 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ -2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3/4 & -1/4 \\ -1/2 & 1/2 \end{pmatrix}\begin{pmatrix} -4 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

2. Find the inverse of this matrix.

$$\begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 2 & -1 \end{pmatrix}$$

Use it to solve this system.

$$\begin{aligned} 2x + \ \ y - z &= 1 \\ y + z &= 1 \\ 2y - z &= 1 \end{aligned}$$

Solution: Here is the inverse.

$$\left(\begin{array}{ccc|ccc} 2 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{-2\rho_2+\rho_3} \left(\begin{array}{ccc|ccc} 2 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -3 & 0 & -2 & 1 \end{array}\right)$$

$$\xrightarrow[(-1/3)\rho_3]{(1/2)\rho_1} \left(\begin{array}{ccc|ccc} 1 & 1/2 & -1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 2/3 & -1/3 \end{array}\right)$$

$$\xrightarrow[-1\rho_3+\rho_2]{(1/2)\rho_3+\rho_1} \left(\begin{array}{ccc|ccc} 1 & 1/2 & 0 & 1/2 & 1/3 & -1/6 \\ 0 & 1 & 0 & 0 & 1/3 & 1/3 \\ 0 & 0 & 1 & 0 & 2/3 & -1/3 \end{array}\right)$$

$$\xrightarrow{(-1/2)\rho_2+\rho_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 1/6 & -1/3 \\ 0 & 1 & 0 & 0 & 1/3 & 1/3 \\ 0 & 0 & 1 & 0 & 2/3 & -1/3 \end{array}\right)$$

To solve the system we have this.

$$\begin{pmatrix} 1/2 & 1/6 & -1/3 \\ 0 & 1/3 & 1/3 \\ 0 & 2/3 & -1/3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1/2 & 1/6 & -1/3 \\ 0 & 1/3 & 1/3 \\ 0 & 2/3 & -1/3 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1/3 \\ 2/3 \\ 1/3 \end{pmatrix}$$