1a) What geometric shape is described by this set?

$$S = \{ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \end{pmatrix} \cdot t \mid t \in \mathbb{R} \}$$

b) Set $t = 2$ to get another point in this set.

c) Use that point to get an alternate description for S (use the letter $\hat{t}$ for the parameter).

Solution:

a) It is a one-parameter linear set, so it is a line.

b) $\begin{pmatrix} 5 \\ 10 \end{pmatrix}$

c) Same slope, just starting at a different place.

$$S = \{ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \end{pmatrix} \cdot \hat{t} \mid \hat{t} \in \mathbb{R} \}$$

2a) Use vector notation to describe set of points (x, y) in the line $y = 2x + 1$.

b) The y-intercept is $(0, 1)$. In the parametrization from the prior part, set $x = 1$ to find another point on the line. Get another description of the same set by replacing the y-intercept with this new point.

Solution:

a) We have this.

$$L = \{ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot x \mid x \in \mathbb{R} \}$$

b) Taking $x = 1$ gives $y = 3$.

$$L = \{ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \hat{x} \mid \hat{x} \in \mathbb{R} \}$$

3. Gauss's method on this system

$$\begin{aligned} x - 2y \phantom{{}-3z} &= 3 \\ 2y - 3z &= 4 \\ 2x - 2y - 3z &= 10 \end{aligned}$$

gives this solution set.

$$\{\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 3/2 \\ 1 \end{pmatrix} \cdot z \mid z \in \mathbb{R}\}.$$

a) Give the associated homogenous system.

b) Describe its solution set using vector notation.

c) Find the solution of the original system when $z = 1$.

d) Use it to give a different description of the solution set.

Solution:

a)

$$\begin{aligned} x - 2y \quad\quad &= 0 \\ 2y - 3z &= 0 \\ 2x - 2y - 3z &= 0 \end{aligned}$$

b)

$$\{\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 3/2 \\ 1 \end{pmatrix} \cdot z \mid z \in \mathbb{R}\}.$$

c)

$$\begin{pmatrix} 7 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 3/2 \\ 1 \end{pmatrix} \cdot 1 = \begin{pmatrix} 10 \\ 5/2 \\ 1 \end{pmatrix}$$

d)

$$\{\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 5/2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 3/2 \\ 1 \end{pmatrix} \cdot z \mid z \in \mathbb{R}\}.$$