When you are working on this material, you must work entirely on your own. You may use the book or your notes. But you may not work with other people, either in the class our outside of it, or use other books or the Internet. If you have any questions, email me.

1. Find the volume of the box determined by these threee vectors.

$$\langle \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \rangle$$

Solution: Take the absolute value of the determinant.

$$\begin{pmatrix} 0 & 3 & 1 \\ 2 & -1 & 2 \\ -3 & 4 & 1 \end{pmatrix} \xrightarrow{\rho_1 \leftrightarrow \rho_2} \begin{pmatrix} 2 & -1 & 2 \\ 0 & 3 & 1 \\ -3 & 4 & 1 \end{pmatrix} \xrightarrow{(3/2)\rho_1 + \rho_3} \begin{pmatrix} 2 & -1 & 2 \\ 0 & 3 & 1 \\ 0 & 5/2 & 4 \end{pmatrix} \xrightarrow{(-5/6)\rho_2 + \rho_3} \begin{pmatrix} 2 & -1 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 19/6 \end{pmatrix}$$

The product down the diagonal is 19, but there was one row swap, so the determinant is -19. The volume is the absolute value of that, $+19$.

2. Calculate the determinant by doing the Laplace expansion down the fourth column.

$$\begin{vmatrix} 2 & 0 & -1 & 2 \\ 1 & 4 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ -3 & 1 & 0 & 1 \end{vmatrix}$$

Solution:

$$\begin{aligned} \begin{vmatrix} 2 & 0 & -1 & 2 \\ 1 & 4 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ -3 & 1 & 0 & 1 \end{vmatrix} &= -2 \cdot \begin{pmatrix} 1 & 4 & 1 \\ 1 & 0 & 0 \\ -3 & 1 & 0 \end{pmatrix} + 0 \cdot \begin{pmatrix} 2 & 0 & -1 \\ 1 & 0 & 0 \\ -3 & 1 & 0 \end{pmatrix} + 1 \cdot \begin{pmatrix} 2 & 0 & -1 \\ 1 & 4 & 1 \\ -3 & 1 & 0 \end{pmatrix} + 1 \cdot \begin{pmatrix} 2 & 0 & -1 \\ 1 & 4 & 1 \\ 1 & 0 & 0 \end{pmatrix} \\ &= -2 \cdot 1 + 0 \cdot (-1) + 1 \cdot (-15) + 1 \cdot 4 \\ &= -13 \end{aligned}$$