To find the roots of a quadratic polynomial, use the quadratic formula.

$$p(x) = a \cdot x^2 + b \cdot x + c \qquad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The *multiplicity* of a root is the number of times it is repeated.

1. Consider this matrix.

$$M = \begin{pmatrix} 1 & 0 & 3 \\ 4 & 2 & 6 \\ 8 & 7 & 9 \end{pmatrix}$$

a) Find the determinant by doing the Laplace expansion along the first row.

Solution:

$$\begin{aligned} \begin{vmatrix} 1 & 0 & 3 \\ 4 & 2 & 6 \\ 8 & 7 & 9 \end{vmatrix} &= 1 \cdot \begin{vmatrix} 2 & 6 \\ 7 & 9 \end{vmatrix} - 0 \cdot \begin{vmatrix} 4 & 6 \\ 8 & 9 \end{vmatrix} + 3 \cdot \begin{vmatrix} 4 & 2 \\ 8 & 7 \end{vmatrix} \\ &= 1 \cdot (18 - 42) - 0 \cdot (36 - 48) + 3 \cdot (28 - 16) \\ &= 1 \cdot (-24) - 0 \cdot (-12) + 3 \cdot (12) = 12 \end{aligned}$$

b) Expand along the second row.

Solution:

$$\begin{aligned} \begin{vmatrix} 1 & 0 & 3 \\ 4 & 2 & 6 \\ 8 & 7 & 9 \end{vmatrix} &= -4 \cdot \begin{vmatrix} 0 & 3 \\ 7 & 9 \end{vmatrix} + 2 \cdot \begin{vmatrix} 1 & 3 \\ 8 & 9 \end{vmatrix} - 6 \cdot \begin{vmatrix} 1 & 0 \\ 8 & 7 \end{vmatrix} \\ &= -4 \cdot (0 \cdot 9 - 3 \cdot 7) + 2 \cdot (1 \cdot 9 - 3 \cdot 8) - 6 \cdot (1 \cdot 7 - 0 \cdot 8) \\ &= -4 \cdot (-21) + 2 \cdot (-15) - 6 \cdot (7) = 12 \end{aligned}$$

c) Expand down the second column.

Solution:

$$\begin{aligned} \begin{vmatrix} 1 & 0 & 3 \\ 4 & 2 & 6 \\ 8 & 7 & 9 \end{vmatrix} &= -0 \cdot \begin{vmatrix} 4 & 6 \\ 8 & 9 \end{vmatrix} + 2 \cdot \begin{vmatrix} 1 & 3 \\ 8 & 9 \end{vmatrix} - 7 \cdot \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} \\ &= -0 \cdot (-12) + 2 \cdot (-15) - 7 \cdot (-6) = 12 \end{aligned}$$

2. For these polynomials, find the roots and their multiplicities. Use the complex numbers, $\mathbb{C}$, where needed.

a) $p_1(x) = (x-5)(x-2)(x+3)$

Solution:

Root	5	2	-3
Multiplicity	1	1	1

b) $p_2(x) = 9 \cdot (x-5)(x+4)$

Solution:

Root	5	-4
Multiplicity	1	1

c) $p_3(x) = 4 \cdot (x-(2/3))(x+3)^2$

Solution:

Root	$2/3$	-3
Multiplicity	1	2

d) $p_4(x) = x^2 - 7x + 10$

Solution: It factors as $(x-5)(x-2)$.

Root	2	5
Multiplicity	1	1

e) $p_5(x) = x^2 + 6x + 9$

Solution: It factors into $(x+3)^2$

Root	-3
Multiplicity	2

f) $p_6(x) = 3x^2 + x + 1$

Solution: The quadratic formula gives two roots.

Root	$\frac{-1+\sqrt{-11}}{6}$	$\frac{-1-\sqrt{-11}}{6}$
Multiplicity	1	1

3. Perform the indicated action in $\mathbb{C}$.

a) $(3+i)(1+i)$

Solution: Start by multiplying as you would if i were a variable, to get this.

$$3 \cdot 1 + 3 \cdot i + 1 \cdot i + i \cdot i = 3 + 4i + (-1) = 2 + 4i$$

Finish by remembering that $i = \sqrt{-1}$, so $i \cdot i = -1$.

$$3 \cdot 1 + 3 \cdot i + 1 \cdot i + i \cdot i = 3 + 4i + (-1) = 2 + 4i$$

b) $(2 - i)(2 + i)$

Solution: Again multiply as you would if i were a variable.

$$2 \cdot 2 + 2 \cdot i + 2 \cdot (-i) + i \cdot (-i)$$

Finish by using that $i \cdot i = -1$.

$$= 4 + 2i - 2i - (-1) = 5$$

c) $(2 + i)^2(1 - i)$

Solution: First combine the first two terms.

$$(2 + i)^2(1 - i) = (4 + 2i + 2i + i^2)(1 - i) = (4 + 4i - 1)(1 - i) = (3 + 4i)(1 - i)$$

Finish by combining the two.

$$(3 + 4i)(1 - i) = 3 - 3i + 4i - 4i^2 = 3 + i - 4(-1) = 7 + i$$