1. Consider the function $f\colon \mathcal{M}_{2\times 2} \to \mathbb{R}^2$ giving this association.

$$f(\begin{pmatrix} a & b \\ c & d \end{pmatrix}) = \begin{pmatrix} a+2c \\ b-d \end{pmatrix}$$

a) What is the dimension of the domain space? The codomain space?

b) Show that the map preserves addition and scalar multiplication.

c) Show that the map preserves linear combinations.

d) Show that the map is not one-to-one.

2. Consider this map $h\colon \mathbb{R}^2 \to \mathbb{R}^4$

$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x-y \\ x \\ 0 \\ x+y \end{pmatrix}$$

a) What are the dimensions?

b) Show that the map is linear.

c) Show that the map is not onto.