First Handin From *Linear Algebra*, by Hefferon

When you are working on this material, you must work entirely on your own. You may use the book or your notes. But you may not work with other people, either in the class our outside of it, or use other books or the Internet. If you have any questions, email me.

1. Find the solution set, using Gauss's method and matrix-vector notation.

$$\begin{aligned} 2x + y \phantom{{}+z} &= 3 \\ 3x - 2y + z &= 1 \\ 7x \phantom{{}-2y} + z &= 7 \end{aligned}$$

Solution:

$$\xrightarrow[-(7/2)\rho_1+\rho_3]{-(3/2)\rho_1+\rho_2} \left(\begin{array}{ccc|c} 2 & 1 & 0 & 3 \\ 0 & -7/2 & 1 & -7/2 \\ 0 & -7/2 & 1 & -7/2 \end{array}\right) \xrightarrow{-\rho_2+\rho_3} \left(\begin{array}{ccc|c} 2 & 1 & 0 & 3 \\ 0 & -7/2 & 1 & -7/2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

The parametrization is this.

$$S = \{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -1/7 \\ 2/7 \\ 1 \end{pmatrix} z \mid z \in \mathbb{R} \}$$

2. Do Gauss-Jordan reduction on this augmented matrix.

$$\left(\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 2 \\ 3 & -1 & 0 & 2 & -2 \\ 2 & 0 & 2 & 1 & 0 \end{array}\right)$$

Solution:

$$\xrightarrow[-2\rho_1+\rho_3]{-3\rho_1+\rho_2} \left(\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 2 \\ 0 & -10 & 0 & -1 & -8 \\ 0 & -6 & 2 & -1 & -4 \end{array}\right)$$

$$\xrightarrow{-(3/5)\rho_2+\rho_3} \xrightarrow[(1/2)\rho_3]{-(1/10)\rho_2} \xrightarrow{-3\rho_2+\rho_1} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 7/10 & -2/5 \\ 0 & 1 & 0 & 1/10 & 4/5 \\ 0 & 0 & 1 & -1/5 & 2/5 \end{array}\right)$$