Writing Mathematics

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In grade school, in high school, and into Calculus, you took classes with people who will not become mathematicians. But starting now, you will be trained as a professional.

Professionals don't write this.

$$0 = 1 \therefore \text{solutions} = \varnothing$$

They instead write this.

> After bringing the system to echelon form, one of the equations is $0 = 1$. Thus the system has no solution.

The person who wrote the first may respond, "You know what I mean." They are inadvertently revealing something: they think of themself as a student, doing the exercises that they've been assigned. In a professional context, it is not for the reader to figure out what you mean, it is for you to say what you mean. Our standard requires writing things that are correct, but we also write them clearly. Learning that is a lifelong pursuit, but here are some first lessons.

All writing has an audience   In here, your audience is your peers. You don't need to justify the quadratic formula or that multiplying by a negative number reverses an inequality, everyone else knows that. But many things they don't know, and those you do need to justify.

Follow the basic rules of rhetoric   Write sentences. Start them with a capital letter, finish them with a period, and in the middle have them follow a reasonable structure.

Where appropriate, break the sentences into paragraphs. The logical structure of your argument may suggest something; for an 'if and only if' proof, you could have an 'if' paragraph and an 'only if' paragraph, or if your proof has several cases then they may make natural breaks, and an induction argument has a base step and an inductive step. In general, if you have more four or five sentences then look for a way to chunk.

The paragraphs provide structure. Math is hard. If you have two paragraphs and the first starts "We begin by . . . " while the second has "We finish by . . . " then you have done a good thing.

In Mathematics, the pronoun is "we" and the tense is present. Don't write "I found the roots," instead say "We next find the roots." Even better, if you are facile enough to pull it off, is to be imperative with "Find the roots."

Symbols   Never start a sentence with a symbol. If you want to say "x is even" and can't think of a better way to avoid it, then write "The number x is even."

When in doubt, use fewer symbols. Prefer "Thus x is even" to "Thus $x = 2k$ for some $k \in \mathbb{N}$," unless you are introducing k to refer to it later. In particular, the only time to use the symbol '$\because$' is in a sentence warning others not to use it. Another suspect is '$\implies$'.

Proving is not teaching   So far, you have seen mathematical presentations only in the context of someone teaching. Proving is different.

Mathematical professionals would not write "At first I tried Sperner's Lemma," as part of a proof, or even "Sperner's Lemma won't work because …" Similarly, you should not say, "The point of dividing by $2x$ is …" If in a document you have reason to include motivation then that's fine, but it goes outside the proof. In this class, we generally won't include such outside material.

Also don't say "The square of an even number is even because for instance $2^2 = 4$ and $10^2 = 100$." Developing intuition with examples is sound teaching but poor proving. Instead, you must provide a general argument along the lines of "The square of an even number is even because $(2k)^2 = 2 \cdot 2k^2$."

That is, in contrast with your past experience with teachers, proof-writers do not feel obliged to say what they are doing. The culture of Mathematics is that, for instance, to prove that if the difference of two numbers is even then so is their sum, a writer may say "If $m, n \in \mathbb{Z}$ are such that $m - n$ is even, then $n + m = (n - m) + 2m$ is the sum of two evens, and is therefore also even." They do not explain how they thought to introduce the $2m$, or what else would work, or whether this extends to multiples of three. Later, when you are working through some tough argument in Abstract Algebra or Real Analysis, you may well think that things would be better if there were more motivation and explanation. That is a conversation for another forum. Today, for professional communication in our field, this is the standard.